

$$\int x f(x) dx =$$

Opener

(A) $x f(x) - \int x f'(x) dx$

(B) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x) dx$

(C) $x f(x) - \frac{x^2}{2} f(x) + C$

(D) $x f(x) - \int f'(x) dx$

(E) $\frac{x^2}{2} \int f(x) dx$

Non-Calculator

$$\int_0^{\frac{\pi}{2}} x \cos x \, dx =$$

- (A) $-\frac{\pi}{2}$ (B) -1 (C) $1 - \frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2} - 1$

$$\int x \sec^2 x \, dx =$$

- (A) $x \tan x + C$ (B) $\frac{x^2}{2} \tan x + C$ (C) $\sec^2 x + 2 \sec^2 x \tan x$
(D) $x \tan x - \ln |\cos x| + C$ (E) $x \tan x + \ln |\cos x| + C$

6-3 day 2 Integration by Parts: Tabular Method

Learning Objectives:

I evaluate an integral using integration by parts
tabular method.

Ex1. Evaluate

$$1.) \int \overset{u}{x^2} \overset{dv}{\cos x} dx$$

$$u = x^2 \quad dv = \cos x \\ du = 2x \quad v = \sin x$$

$$= x^2 \sin x - \int \sin x \cdot 2x dx$$

$$= x^2 \sin x - \int 2x \sin x dx$$

$$u = 2x \quad dv = \sin x \\ du = 2 \quad v = -\cos x$$

$$= x^2 \sin x - \left[2x \cdot (-\cos x) - \int (-\cos x) \cdot 2 dx \right]$$

$$= x^2 \sin x - \left[-2x \cos x + \int 2 \cos x dx \right]$$

$$= x^2 \sin x + 2x \cos x - 2 \int \cos x dx$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C$$

DER	A.D
$+ x^2$	$\cos x$
$- 2x$	$\sin x$
$+ 2$	$-\cos x$
0	$-\sin x$

$$x^2 \sin x + 2x \cos x - 2 \sin x + C$$

2.) $\int x^3 e^x dx$

← DER → A.D.

$u = x^3$
 $du = 3x^2$

$dv = e^x$
 $v = e^x$

$x^3 e^x - \int 3x^2 e^x dx$

$x^3 e^x - (3x^2 e^x - \int 6x e^x dx)$ $u = 3x^2$ $du = 6x$

$dv = e^x$
 $v = e^x$

$x^3 e^x - (3x^2 e^x - (6x e^x - \int 6e^x dx))$

$u = 6x$ $du = 6$ $dv = e^x$
 $v = e^x$

$+ x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$

(Note: In the original image, the terms x^3 , $3x^2$, $6x$, and $6e^x$ are circled in red, and the e^x parts are circled in blue. Below each term are labels: f AD, f' AD, f'' , and f''' AD.)

DER	A.D.
+ x^3	e^x
$- 3x^2$	e^x
$+ 6x$	e^x
$- 6$	e^x
0	e^x

$$x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C$$

3.) $\int x^5 e^{-2x} dx$

DER	Anti
+ x ⁵	e^{-2x}
- 5x ⁴	$\frac{-1}{2} e^{-2x}$
+ 20x ³	$\frac{1}{4} e^{-2x}$
- 60x ²	$\frac{-1}{8} e^{-2x}$
+ 120x	$\frac{1}{16} e^{-2x}$
- 120	$\frac{-1}{32} e^{-2x}$
0	$\frac{1}{64} e^{-2x}$

$$= x^5 \left(\frac{-1}{2} e^{-2x} \right) - (5x^4) \left(\frac{1}{4} e^{-2x} \right) + (20x^3) \left(\frac{-1}{8} e^{-2x} \right) - (60x^2) \left(\frac{1}{16} e^{-2x} \right) + (120x) \left(\frac{-1}{32} e^{-2x} \right) - 120 \left(\frac{1}{64} e^{-2x} \right)$$

$$= -\frac{1}{2} x^5 e^{-2x} - \frac{5}{4} x^4 e^{-2x} - \frac{5}{2} x^3 e^{-2x} - \frac{15}{4} x^2 e^{-2x} - \frac{15}{4} x e^{-2x} - \frac{15}{8} e^{-2x} + C$$

4.) $\int e^x \cos x dx$ $u = \cos x$ $dv = e^x$
 $du = -\sin x$ $v = e^x$

$$\int e^x \cos x dx = e^x \cos x - \int -e^x \sin x dx$$

$$\int e^x \cos x dx = e^x \cos x + \boxed{\int e^x \sin x dx}$$

$u = \sin x$ $dv = e^x$
 $du = \cos x$ $v = e^x$

$\int e^x \cos x dx = e^x \cos x + e^x \sin x - \int e^x \cos x dx$
 $+ \int e^x \cos x dx$ $+ \int e^x \cos x dx$

$$\int e^x \cos x dx = \frac{e^x \cos x + e^x \sin x}{2}$$

$$\int e^x \cos x dx = \frac{e^x \cos x + e^x \sin x}{2} + C$$

Homework

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